Experiment OD1: **Diffraction and Interference Intensity**

Theoretical Background: In 1690 Huygens published his *Treatise on Light* in which he suggested that light is a wave moving through an all-pervading ethereal matter. In his 1704 text *Optiks*, Isaac Newton argued against this wave picture and proposed that light was composed of small material bodies or particles. Newton's view was widely accepted for nearly a century until, in 1803, Thomas Young performed a double-slit experiment, demonstrating interference effects and establishing the validity of the wave nature of light. In 1864 Maxwell provided the theoretical foundation for this wave picture of light. Using the traveling wave solutions to Maxwell's equations one can compute the interference pattern produced when a plane wave is incident on a set of narrow rectangular slits. In this lab, you will measure the intensity of several multi-slit interference patterns and fit these data to the theoretical model provided by Maxwell. Such fitting allows one to extract quantitative physical information about the source of the interference (i.e., size and spacing of the slits). This approach is, in principle, identical to the procedure used to determine crystal structure using X-rays.

Experimental Apparatus: Our light source will be a Helium-Neon laser used in conjunction with the Pasco light sensor and Smart Pulley. Data from the latter two sensors will be collected through a Data Studio computer interface. Both the gain (i.e., amplification) and sampling rate of the light sensor can be adjusted. A variable speed motor will be provided to move the light sensor (mounted on an adjustable friction cart) along a track at a very slow and relatively steady pace. You will need to design an experimental setup that accomplishes this task. You will have multiple sets of slits available to produce several different patterns to investigate, compare, and contrast. A pair of linear polarizers and a neutral filter will also be available to reduce the intensity of the beam.

Procedure: You can set up the laser, slits, and possible polarizer(s) on a table on one side of the room. Jacks will be provided to allow you to set the height of the beam and level the interference pattern so that the light sensor intercepts the beam all the way across the track. (This leveling is critical and is best assessed by collecting interference data ... the pattern needs to be symmetric about the center). You will need to determine a way for the Smart Pulley to give you information about the position of the cart and light sensor vs. time so that this information can later be coordinated with the output of the light sensor. You may wish to explore how the resolution of the light sensor affects the resolution of your data, and how you could improve that.

Start by taking data for some single and double slits of known dimensions. Analyze this data to both learn the analysis procedure and to verify your setup. Then take data for the four "unknown" slits in the "Advanced Lab" slide. A primary goal of this experiment is a quantitative determination of the relevant dimensions of these unknown slits. Finally, measure the interference pattern produced by the mounted thin wires and by a human hair and analyze these pattern (using Babinet's principle) to obtain the wire and hair diameters. Note that you must discard the forward scattering data (i.e., $\theta = 0$ region) in this analysis.

Data Analysis: Remember that for this lab you will have both an oral exam and prepare a formal lab report (see syllabus for dates). For your presentation and lab report you should understand the exact formula which predicts the interference pattern. You will use Kaleidagraph to make plots of position (on the horizontal axis) versus intensity, and use its curve fitting capabilities to get the best fit to the data. The exact intensity equation can be used for the fit with the parameters involving the slit spacing, slit width, an amplitude, and possibly an additive term which sets a background level. Details on how to carry out such a multi-parameter curve fit are given below. Once you obtain a good curve fit, you should prepare a plot that includes the fit as a continuous line and your original data as symbols. Be aware of the approximations used in the derivation of the interference equation and the role they play in your experimental analysis. Please note that both the "raw data" and the final fitted data must be included in your notebook and paper.

Additional Data Analysis Information:

Data from the two sensors will be of the form $x(t_1)$ (position from the smart pulley) and $I(t_2)$ (intensity from the light sensor). [This raw data is stored in separate data tables with columns: time, position and time, intensity]. We want to model the function I(x) so we need to merge these two data sets. Unfortunately, the sensors sample at different rates so the time columns will not be identical, thus we need to interpolate the two data sets. This interpolation and the data analysis will be done using the program KaleidaGraph.

You first need to copy your four data columns into KaleidaGraph. By default, this program will read in your data as "text" so, after pasting in your data, you will need to change the column format to "float". If your data is entered as follows:

t_1 [c0] $x(t_1)$ [c1] t_2 [c2] $I(t_2)$ [c3]

where the numbers in square braces are the KG column numbers, you can create a column of $x(t_2)$ values (i.e., interpolated position values based on your intensity time values) using the following formula (entered in the "Formula" window):

c4 = table(c2, c0, c1)

so your final KG spreadsheet should have the form:

t_1 [c0] $x(t_1)$ [c1] t_2 [c2] $I(t_2)$ [c3] $x(t_2)$ [c4].

You can now plot $I(t_2)$ versus $x(t_2)$ (use the "scatter plot" option from the "Gallery" menu). For the data analysis you will want to shift your x-values so that x=0 defines the center of the interference pattern. You can do this through the formula window as follows:

$c4 = c4 - x_{center}$

where x_{center} is the original x-value at the center of your pattern.

Now you will attempt to fit these data to our theoretical expression for I(x). We will do this using the multi-variable non-linear curve fitting capabilities of the KG program. You will need to add and define a new general curve fit function. To add a menu entry for a new "General" curve fit: select "Edit General" and Add "New Fit", highlight "New Fit" in the left window and rename this item. Now you can define your fit. For this you must type in the theoretical expression for I(x) using the variable m0 for x and the variables m1, m2, m3, etc. for the "fitting" parameters. You must also specify starting values for the fit parameters after the formula. All of these entries are separated by semi-colons. Finally, be sure that radians is selected in the Curve Fit Definition window.

For example if you wanted to specify a quadratic fit $y(x) = ax^2 + bx + c$ and you know that a≈1, b≈0.3, and c≈7, you would enter:

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m1*m0^2 + m2*m0 + m3;  m1=1.0;  m2=0.3;  m3=7.0;
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For this fit, KaleidaGraph will search through a 3-dimensional parameter space of (a,b,c)=(m1,m2,m3) values, starting from your initial values, trying to minimize the "distance" between your data set and the theoretical function. This multi-dimensional fitting can be very tricky since the (a,b,c) parameter space may have many "local minima" which may "trap" the search routine. Thus a good starting estimate of the fit parameters is generally needed to locate the "global" minimum (i.e., the true best fit) for your function. KaleidaGraph uses the Levenberg-Marquardt algorithm for this fitting.

For your I(x) data analysis your fitting parameters should include slit width, slit spacing, I_{max}, and possibly an additive $I_{background}$ term. You will want fits for both the central part of the interference pattern (taken with a reduced intensity beam) and for the "wings" (taken with the full intensity beam which saturates the detector in the center of the pattern). KG will return "best fit" parameters with uncertainty estimates as well as a plot of the best fit function. For a good representation of this function you should specify at least 1000 curve fit points from the "Curve Fit Options" menu.

Finally, you need to understand the meaning of χ^2, which is the standard measure of "goodness of fit" and is the function that the fitting routine attempts to minimize. For a set of data points $\{x_i, y_i \pm \sigma_i\}$, where σ_i is the uncertainty in the measured value of y_i, and a fitting function $y = f(x)$, χ^2 is defined as

$$\chi^2 = \sum_{i=1}^{N} \frac{[y_i - f(x_i)]^2}{\sigma_i^2}.$$

For a "good" fit we expect $\chi^2 \approx N$ (a fact that you should be able to explain). By default, KaleidaGraph sets σ_i = 1 for all i, so you either need to specify σ_i for your intensity data set, or assuming σ_i is approximately the same for each measurement (i.e., $\sigma_i = \sigma$ for all i), simply compare the computed default χ^2 to $N\sigma^2$.

An Additional Measurement ... Diffraction due to a Straightedge

A single straightedge also produces an interference effect. To see this place a short focal length diverging lens close to the laser and a longer focal length diverging lens a short distance away to produce a large illuminated region across the photo-detector track. Now place a vertical straightedge (e.g., a razor blade) in the light path between the two lenses to block out half of the illuminated region. You should be able to observe an interference pattern. Measure the light intensity across this pattern (including across the shadow region). Also, for a reference, measure the intensity across the entire region without the straight edge (being careful not to disturb the lenses when you remove the straightedge). [You may want to turn up the gain on the photo-detector for these measurements.]

Plot your results to overlay the two data sets. Do you get any light in the straightedge "shadow region"? How do the maximum intensities with and without the straightedge compare?