Physics 330 **Fall (3) 2016**

Analytical Mechanics: Problem Set #1

Math Methods and Newtonian Mechanics

$$h_i = \left|\frac{\partial \vec{r}}{\partial u_i}\right| \ ; \ \hat{e}_i = \frac{1}{h_i}\frac{\partial \vec{r}}{\partial u_i} \ ; \ \frac{d\vec{A}}{ds} = \sum_i \left(\frac{dA_i}{ds}\hat{e}_i + A_i \frac{d\hat{e}_i}{ds}\right) \ ; \ \vec{F}_{ext} = \frac{d\vec{p}}{dt} = m\frac{d\vec{v}}{dt} + \vec{u}\frac{dm}{dt} \ ; \ \vec{F} = -\vec{\nabla}U$$

<u>Due: Sunday Dec. 4 by 5 pm</u>

Reading assignment: for Wednesday, 1.1-1.5, 1.8-1.16 (vectors and coordinate systems)
for Thursday, 2.1-2.4 (equations of motion from Newton's 2nd law)
for Friday, 2.4-2.7 (retarding forces, conservation laws, rockets)

Overview: We will begin our study of analytical mechanics with a brief review of vectors and associated math methods. We then launch into the Newtonian approach to classical mechanics. The "Newtonian program" is summarized in Newton's second law $\vec{F} = m\vec{a}$ (or, more correctly, as given above). A determination of the forces gives the accelerations; velocities and positions are then computed via integration. Newton's laws completely describe all of the phenomena of classical mechanics and, at this point in your careers, I'm sure you all are intimately familiar with them. What you may not be aware of is that the Newtonian program is not the most general formulation of classical mechanics and is not always the most convenient approach. We will be learning other quite elegant formulations of classical mechanics in this course. However, there are still many cases where good ole $\vec{F} = m\vec{a}$ remains the method of choice. Note that all formulations of mechanics, from the pedestrian to the polished, lead to the same equations of motion. Unfortunately, these differential equations are often intractable and thus one must resort to perturbation techniques or numerical methods. You should get used to turning to the computer as a tool in solving problems for this class. One can only get so far with analytic techniques and the "real" world is a complicated place! But never fear, I won't deny you the delight of elegant analytical solutions. Note however that even for these, it is often quite instructive to make plots of your final results to get a better feeling for the physical behavior described by your equations.

In-Class Problems:

Wed.
- C01.1 (vector manipulations) [1 part each]
- C01.2 (an elliptic orbit) [CO]
- C01.3 (circular motion) [TP]
- C01.4 (angles in a cube) [JS]
- C01.5 (vector decomposition) [CV]
- C01.6 (law of cosines) [JR]
- C01.7 (analysis of a surface z(x,y)) [JW]
- C01.8 (perpendicular vectors) [everyone]

Thurs.
- C02.1 (shooting uphill) [JS and JW]
- C02.2 (two blocks and an inclined plane with friction) [CO and JR]
- C02.3 (block launched from a track) [TP and CV]

Fri.
- C03.1 (rise and fall of a particle in drag) [JR and CV]
- C03.2 (boat subject to an exponential drag force) [CO and JS]
- C03.3 (falling raindrop) [TP and JW]

(over for problem assignment)

Problem Assignment (10 total plus bonus):

1.1 Show that the unit vectors in spherical coordinates are given by:

$$\hat{e}_r = \hat{x}\cos\varphi\sin\theta + \hat{y}\sin\varphi\sin\theta + \hat{z}\cos\theta$$
$$\hat{e}_\theta = \hat{x}\cos\varphi\cos\theta + \hat{y}\sin\varphi\cos\theta - \hat{z}\sin\theta\ ,$$
$$\hat{e}_\varphi = -\hat{x}\sin\varphi + \hat{y}\cos\varphi\ .$$

1.2 A particle moves at constant speed v around a closed path given by $r = k(1+\cos\theta)$. Show that the magnitude of the particle's acceleration is

$$a = \sqrt{a_r^2 + a_\theta^2} = \frac{3v^2}{4k}\sqrt{\frac{2}{1+\cos\theta}}$$

[Some important intermediate results are $\dot{\theta} = v(2kr)^{-1/2}$ and $\ddot{r} = -v^2/(4k)$].

1.3 You hit a softball at a height of 0.70 m above home plate. The ball leaves your bat at an angle of 37º above the horizontal and travels towards a 2.0 m tall fence, 60 m away in center field. What initial speed does the ball need in order to just clear the fence? [Ignore drag]

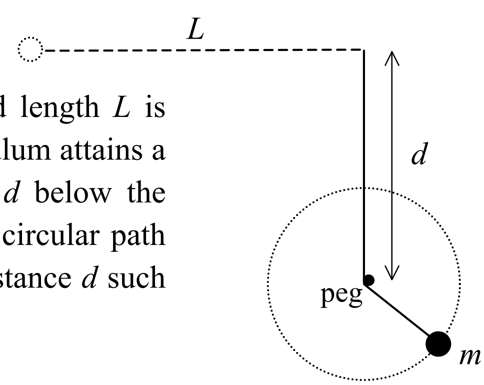
1.4 Use Newton to re-analyze problem 1.2 with a drag force given by $F_D = \kappa\rho A v^2$ where $\kappa=0.25$, the air density at STP is $\rho=1.29$ kg/m^3, and the ball's mass and radius are 200 g and 5.0 cm, respectively. You need to submit annotated plots from Newton. Find the required speed to within 0.1 m/s.

1.5 A particle of mass m moves in a medium with a drag force equal to $mk(v^3 + a^2 v)$, where k and a are constants. Show that for any value of the initial speed the particle will never move a distance greater than $\pi/2ka$ and that the particle only comes to rest in the infinite time limit.

1.6 A particle of mass m is subjected to a one-dimensional, time-dependent force $F(t) = kte^{-bt}$, where k and b are constants. If the particle is initially at rest determine the position, speed, and acceleration of the particle as a function of time. Plot these results for $0 \le t \le 20$ s for the case of $m = 1.0$ kg, $k = 1.0$ N/s, and $b = 0.5$ s^{-1}.

1.7 Superball and the marble: A superball of mass M and a marble of mass m are dropped from a height of h with the marble just on top of the superball. (A superball has a coefficient of restitution of nearly 1 so it undergoes essentially elastic collisions with hard surfaces). The superball collides with the floor, rebounds, and smacks the marble which moves back up. (a) How high does the marble go if all the motion is vertical? (b) How high does the superball go? (Ignore the sizes of the superball and marble).

1.8 Peg and the pendulum: A pendulum with mass m and length L is released from rest from a horizontal position. When the pendulum attains a vertical orientation, it encounters a peg, located a distance d below the pivot point. This peg causes the pendulum to move along a circular path (of radius $L-d$) as shown in the figure. Find the minimum distance d such that the pendulum will swing completely around the circle.



1.9 Two hobos, each of mass m_h, are standing at one end of a stationary railroad flatcar with frictionless wheels and mass M_c. Each hobo can obtain a speed u (relative to the car) running to the other end of the car. (a) Use conservation of momentum to find the final speed of the car if the two hobos run together and jump simultaneously. (b) Repeat this calculation for the case where first one hobo runs and jumps and then the second runs and jumps. Which case gives the larger final speed? (Note that the speed u is relative to the car and is independent of the car's motion).

1.10 Consider a single stage rocket taking off from Earth. (a) Show that the height of the rocket at burnout is given by

$$y_b = ut_b - \frac{1}{2}gt_b^2 - \frac{mu}{\alpha}\ln\left(\frac{m_o}{m}\right) \quad \text{where} \quad m = m_o - \alpha t_b.$$

(b) How much farther in height will the rocket go after burnout?

Bonus: Use the results of problem 1.1 to show that the components of the acceleration vector in spherical coordinates are given by:

$$\begin{aligned}
a_r &= \ddot{r} - r\dot{\theta}^2 - r\sin^2\theta\dot{\varphi}^2 \\
a_\theta &= 2\dot{r}\dot{\theta} + r\ddot{\theta} - r\sin\theta\cos\theta\dot{\varphi}^2 \\
a_\varphi &= 2\dot{r}\sin\theta\dot{\varphi} + 2r\cos\theta\dot{\theta}\dot{\varphi} + r\sin\theta\ddot{\varphi}
\end{aligned}$$