Physics 375 **Spring (12) 2003**

Solid State Physics: Problem Set #9

Electronic Properties of Solids: Periodic Potential and Band Structure

Due: Friday Mar. 21 by 6 pm

Reading assignment: for Monday, 7.3-7.5 (Brillouin zones and energy bands)
for Wednesday, 7.6-7.7 (nearly free electron approx. and empty lattice model)
for Friday, 7.8-7.12 (band filling in the elements)

Problem assignment:

Chapter 7 Problems:

*7.2 When Fermi sphere meets zone face **[Brian]**
*7.5 Density of states in the "tight-binding" model **[Robert]**
7.11 Brillouin zones for a rectangular lattice

A1. *Penetration depth for conductors*. When an electromagnetic wave enters a conducting material the wave is strongly attenuated if the frequency of the wave ω is less than the plasma frequency of the conductor ω_p. The penetration or skin depth of a conductor $\delta(\omega)$ is defined as the distance over which the wave amplitude decreases by a factor of 1/e.
a) Show that in the case when $\omega\tau$<<1 the penetration depth is given by

$$\delta(\omega) = \sqrt{\frac{c^2\epsilon_o}{\omega\sigma_o}}$$

where σ_o is the DC conductivity of the material.
b) Estimate $\delta(\omega)$ for silver for visible radiation. (Use the Drude model to determine σ_o for silver).
c) Make a log-log plot of $\delta(\omega)$ vs. ω for silver for the frequency range: $10^5 < \omega < 10^{13}$ rad/s.
d) The DC conductivity of seawater is about 5 $\Omega^{-1}\text{m}^{-1}$. Determine the penetration depth of radio waves into the ocean and comment on the feasibility of radio communication with submarines.

A2. *Simple Kronig-Penney model (aka the Dirac comb)*. The electronic band structure of solids is due to the periodic nature of the potential felt by the valence electrons. Here we will show that this type of band structure shows up in the simplest possible model of a periodic potential. Consider a one-dimensional lattice with spacing a where the potential at each lattice site is given by a Dirac-delta-function "well" $-\alpha\delta(x)$ where α measures the "depth" of the well. For N such δ-function wells the total potential is

$$V(x) = -\alpha\sum_{j=0}^{N-1}\delta(x-ja)$$

Since this potential is of the form $V(x+a)=V(x)$, solutions of Schrodinger's equation must be of the Bloch form: $\psi(x+a)=e^{iKa}\psi(x)$ where K is a constant.

a) By assuming periodic boundary conditions $\psi(x+Na) = \psi(x)$ show that $K=2\pi n/Na$ where n = 0, ±1, ±2, ... (Since N is assumed very large, K is essentially a continuous variable).

b) Defining $k = \sqrt{2mE}/\hbar$, show that the general solution to the Schrodinger equation in the first cell to the right of the origin can be written as

$$\psi_R(x) = A\sin(kx) + B\cos(kx) \qquad 0<x<a$$

and thus, from Bloch's theorem, the solution in the first cell to the left of the origin must be

$$\psi_L(x) = e^{-iKa}\,[A\sin k(x+a) + B\cos k(x+a)] \qquad -a<x<0.$$

c) The boundary conditions on $\psi(x)$ associated with the δ-function potential at $x=0$ are as follows:

$$\psi_R(0) = \psi_L(0) \quad ; \quad \psi_R'(0) - \psi_L'(0) = -\frac{2m\alpha}{\hbar^2}\psi(0)$$

Use these conditions to eliminate the constants A and B and show that allowed energy states of the system are given by the equation:

$$\cos(Ka) = \cos(ka) - \left(\frac{m\alpha a}{\hbar^2}\right)\frac{\sin(ka)}{ka}$$

Note that the left-hand side of this equation is bounded by ±1 and thus, if the right-hand side falls out of the range (–1,1) no solution for k is possible. These gaps in the solution represent forbidden energies. They are separated by bands of allowed energies.

d) Plot the right-hand side of the above energy equation vs. ka for $0 \le ka \le 16$ letting $m\alpha a/\hbar^2 = 5$. Indicate the forbidden regions of ka on your plot and use the Maple function fsolve( ) to determine the precise ka values locating the bottom and top of each band found in your plot.

*To be presented in class on Friday.