

Solid State Physics: Problem Set #8  
**The Free Electron Model:  
 Transport and Optical Properties of Metals**

Due: Friday Mar. 7 by 6 pm

Note: Write-up for experiment #3 (Bragg X-ray) is now due on Friday Mar. 7.

Reading assignment: for Monday, 6.4, 6.7 (Drude model for electrical and thermal transport)  
 for Wednesday, 6.5 (optical properties of metals)  
 for Friday, 7.1-7.2 (the periodic potential and energy gaps)

Problem assignment:

Chapter 6 Problems:

- 6.11 Wavelength at which Al becomes transparent
- 6.15 Plasma frequency and "optical mass" for Al from  $\epsilon(\omega)$  data
- \*6.16 Loss of reflectivity for Hg **[Melissa]**
- \*6.20 Total internal reflection of X-rays in Si **[Peter]**

A1. *Static magnetoconductivity tensor.* As shown in the text (pg. 164), the steady state solution to the Drude model for the free electron gas in an external magnetic field is given by

$$\vec{v} = -\frac{e\tau}{m}(\vec{E} + \vec{v} \times \vec{B}).$$

a) Show that for  $\vec{B} = B\hat{z}$ , the static current density  $\vec{j} = -ne\vec{v}$  can be written in the form  $\vec{j} = \hat{\sigma}\vec{E}$ , where  $\hat{\sigma}$  is the conductivity tensor, as follows

$$\begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix} = \frac{\sigma_0}{1 + \omega_c^2 \tau^2} \begin{pmatrix} 1 & -\omega_c \tau & 0 \\ \omega_c \tau & 1 & 0 \\ 0 & 0 & 1 + \omega_c^2 \tau^2 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}$$

where  $\omega_c = eB/m$  is called the cyclotron frequency.

b) In the high field limit of  $\omega_c \tau \gg 1$ , show that  $\sigma_{yx} = -\sigma_{xy} = nec/B$ . These off-diagonal elements are known as the Hall conductivity.

\*To be presented in class on Friday.