# Solid State Physics: Problem Set #6

## Thermal Properties of Solids II: Vibrational Modes and Phonons

Due: Friday Feb. 21 by 6 pm

Note: You should try to get experiment #3 (Bragg X-ray) completed this week.

Reading assignment: for Monday, 5.5-5.7 (acoustic and optical modes)
for Wednesday, 5.9-5.12 (phonons and Brillouin zones)
for Friday, 6.1 (the free electron ideal gas model)

Problem assignment:

Chapter 5 Problems:

5.5 Exact frequency spectrum $N(\omega)$ for the 1-D lattice chain
(Also prove that the total number of modes is $N$)

*5.6 1-D lattice chain with second nearest neighbor coupling **[Melissa]**

5.8 Acoustic vs. optical modes in the diatomic lattice chain

*5.10 Exact dispersion relation $\omega(k)$ for the 2-D triangular lattice **[Peter]**

5.15 Experimental dispersion relation for Al in [200] and [220] directions

A1. Vibrational energy in the 1D lattice chain: Consider a longitudinal wave of amplitude $u$

$$u_s = u\cos(ksa - \omega t)$$

which propagates in a monatomic linear lattice of $N$ identical atoms of mass $M$, spacing $a$, and nearest neighbor force constant $c_1$.

(a) Show that the total energy of the wave is

$$E = \tfrac{1}{2}M\sum_s (du_s/dt)^2 + \tfrac{1}{2}c_1\sum_s (u_s - u_{s+1})^2$$

where the index $s$ runs over all atoms.

(b) By substituting $u_s$ into the part (a) expression, show that the time-averaged total energy per atom is

$$\langle \varepsilon \rangle = \frac{1}{T}\int_0^T \varepsilon(t)dt = \tfrac{1}{4}M\omega^2u^2 + \tfrac{1}{2}c_1(1-\cos ka)u^2 = \tfrac{1}{2}M\omega^2u^2$$

where $\varepsilon = E/N$ and, for the final step, you use the Eq. 5.18 dispersion relation.

*To be presented in class on Friday.