

## Solid State Physics: Problem Set #5

**Thermal Properties of Solids I:  
Einstein and Debye Models**

Due: Friday Feb. 14 by 6 pm

Note: Write-up for experiment #2 (Laue X-ray) due on Friday Feb. 14.

Reading assignment: for Monday, 5.2, 5.5-5.6 (density of modes and the Debye model)  
for Wednesday, No class (Mark out of town)  
for Friday, 5.3-5.4 (harmonic approximation for  $\rho(k)$ )

Problem assignment:

Chapter 5 Problems:

\*5.2 High and low temperature limits for Debye model **[Brian]**

\*5.3 Debye model for a two-dimensional solid **[Robert]**

A1. *Einstein Model:* In 1906, Einstein had available the following data for the specific heat of diamond:

$T(K)$	$C/3Nk_B$	$T(K)$	$C/3Nk_B$
222.4	0.1278	413.0	0.4463
262.4	0.1922	479.2	0.5501
283.7	0.2271	520.0	0.6089
306.4	0.2653	879.7	0.8871
331.3	0.3082	1,079.7	0.9034
358.5	0.3552	1,258.0	0.9235

Graph these data along with Einstein's theoretical approximation. You will need to determine a value for the Einstein temperature  $\Theta_E$  of diamond. You can do this by forcing agreement at one temperature (which Einstein did) or by fitting the entire data set to the theoretical equation (which is what Einstein would have done if he had had a computer!).

A2. *Debye temperature for diamond:* Determine  $\Theta_D$  for diamond given the lattice constant for this material is 3.56 Å (for the conventional cubic cell, see Fig. 2.12) and that the speed of sound in diamond is 12,000 m/s. Also determine the wavelength corresponding to the Debye cutoff frequency  $\omega_{max}$  and compare to this the inter-atomic spacing in diamond. [answer:  $\Theta_D=2010$  K]

A3. (a) Show that the result obtained in problem 5.2 for the Debye specific heat can be written as follows:

$$\frac{C_D}{3Nk_B} = 3 \left( \frac{T}{\Theta_D} \right)^3 \int_0^{\Theta_D/T} dz \frac{z^4 e^z}{(e^z - 1)^2}$$

(b) Use Maple to numerically evaluate this integral for  $T/\Theta_D = 0.1, 0.2, 0.5, 1.0,$  and  $2.0$ .

(c) Find the temperature ( $T/\Theta_D$ ) at which the  $T^3$  approximation for  $C_D$  (Eq. 5.9) is in error by 5%.

(d) Using the Debye temperature for diamond found in A2, numerically determine  $C_D/3Nk_B$  for some of the temperatures given in the problem A1 data table. How do they compare with  $C_E/3Nk_B$ .

\*To be presented in class on Friday.