Thermal Physics: Problem Set #8

Quantum Statistics II (Photons and Ideal Boson Systems)

$$\bar{n}_{Pl}(f)=\frac{1}{e^{hf/kT}-1}\ ;\ D_{EM}(f)=\frac{8\pi}{c}Vf^2\ ;\ \frac{dU_{rad}}{dt}=\sigma eAT^4\ ;\ \bar{n}_{BE}(\varepsilon)=\frac{1}{e^{(\varepsilon-\mu)/kT}-1}\ ;\ g(\varepsilon)=2\pi g_s\left(\frac{2m}{h^2}\right)^{3/2}V\sqrt{\varepsilon}$$

Due: Friday Feb. 20 by 6 pm

Reading Assignment: for Mon, 7.5 (7 pgs) (Phonons and the Debye theory of solids)
for Wed, 7.6, 5.3 (15 pgs) (Bose-Einstein condensation & Phase transitions)
for Fri, 5.3 (18 pgs) (Phase transitions continued)

Overview: Electromagnetic radiation in a cavity and sound waves in a solid can each be described in terms of a superposition of fundamental or normal modes. Each mode corresponds to a harmonic oscillator-like term in the appropriate Hamiltonian and energy quantization leads to the particle-like photon or phonon description of these modes. (The case of phonons is *the* classic example of a quasi-particle description of the collective excitations in a multi-body system). The energy in each mode (i.e., frequency f) is $E = nhf$ where n is the number of "particles" in that mode and the sum over all occupation numbers is a geometric series in $x = e^{-\beta hf}$ giving $Z(f, T) = 1/(1-x)$. The total particle number is not fixed in these systems as "particles" are created and destroyed with energy fluctuations and variations in the temperature (and the ground state has zero particles!). To deal with systems with variable particle number we generally work in the grand-canonical ensemble where particle number is controlled by the chemical potential μ. Comparing the number distribution function for photons with the Bose-Einstein distribution we conclude that $\mu = 0$ for these $m = 0$ quasi-particles. Combining $n(f)$ with a density of modes $D(f)$ allows for calculation of all thermodynamic properties. $D_{EM}(f)$ is readily computed for radiation in a cavity and the famous Planck spectrum directly follows. In the case of vibrations or phonons in a solid we must construct a microscopic model in order to determine $D(f)$. The models proposed by Einstein and Debye assume different $D(f)$ and thus give different thermodynamic behavior. Low temperature specific heat data show the Debye model to be the more realistic of the two. An ideal gas of $m \neq 0$ bosons is characterized by a ground state with all particles in the lowest energy single-particle state. However, this ground state is approached in a rather unusual fashion. At a temperature well above $T = 0$ the ideal boson system undergoes a "condensation" transition in which there is a macroscopic occupation of the lowest energy single-particle state (something you might think would be entropically forbidden ... but such are the ways of quantum weirdness!) Although this condensation transition was predicted by Einstein in 1924, it was not until 1995 (and after years of effort) that it was actually observed experimentally.

Problem Assignment: (9 problems total)

- **7.37** (peak of the Planck spectrum)
- **7.38** (temperature dependence of the Planck spectrum)
- **7.39** (wavelength dependence of the Planck spectrum)
- **7.43** (photons emitted by the sun)
- ***7.44** (number of photons in a hot box) **[Christian]**
- **7.51** (thermal emission of a light bulb)
- ***7.61** (Debye model for liquid ^{4}He C_v) **[Ryogo]**

*denotes a problem to be presented in class

***A1) <u>Black hole thermodynamics and baby black holes</u>** **[Su]**

Black holes are the ultimate blackbodies. According to Stephan Hawking the entropy of a black hole of mass M is given by $S(M) = 8\pi^2 GM^2 k/hc$. Using Einstein's relation $E=Mc^2$ we can express this entropy as a function of energy $S(E)$ and thus a temperature can be defined via thermodynamics. Assuming black holes radiate according to the Stephan-Boltzmann law, construct an equation for the rate of mass loss dM/dt due to this "Hawking" radiation. Solve this to determine $M(t)$ and compute the "lifetime" of a baby black hole (give some numerical results in the range $1\text{ kg} \le M_o \le M_{earth}$).

[See Hawking, Sci. Am. **236**, 34 (1977) and Beckenstein, Physics Today **33**, 24 (1980)]

A2) <u>Debye model for copper - Numerical results</u>

(a) Starting from the Debye model result for U, given in Eq. 7.109, show that the Debye heat capacity $C_v = \partial U/\partial T$ can be written as follows:

$$C_V = 9Nk\left(\frac{T}{T_D}\right)^3 \int_0^{T_D/T} \frac{x^4 e^x}{(e^x - 1)^2}\, dx$$

(b) Use Maple's Int() function to numerically evaluate this integral for $T/T_D = 0.1, 0.2, 0.5, 1.0$, and 2.0.

(c) Determine the Debye temperature for copper using the following data: ρ=8.93 g/cm^3, c_s=2,325 m/s.

(d) Use Maple to compute and plot the Debye heat capacity (C_v/Nk) versus temperature T for copper $0 < T < 500$K. Also include the T^3 low temperature approximation on your plot and make note of the temperature range over which this approximation appears to work.