Thermal Physics: Problem Set #4

Paramagnetism, Mechanical & Diffusive Equilibrium, Heat Engines

$$M = N\mu_m \tanh\left(\frac{\mu_m B}{kT}\right) \; ; \; P = T\left(\frac{\partial S}{\partial V}\right)_{U,N} \; ; \; \mu = -T\left(\frac{\partial S}{\partial N}\right)_{U,V} \; ; \; dU = TdS - PdV + \mu dN \; ; \; e = \frac{W}{Q_h} \; ; \; e \le 1 - \frac{T_c}{T_h}$$

Due: Friday Feb. 6 by 6 pm

Reading Assignment: for Mon, 3.5,4.1-4.2 (16 pgs) (Diffusive equilibrium, Carnot cycle)
for Wed, 4.3-4.4 (10 pgs) (Heat Engines and refrigerators)
for Fri, 4.4 (7 pgs) (Liquefaction of gases and low temperatures)

Overview: This week we extend our study of entropy and equilibrium to examine systems with both exotic and extremely practical behaviors. The two-state paramagnet (a collection of non-interacting spin 1/2 particles) is a model that can be solved analytically and studied experimentally. This system has a finite maximum energy, a requirement to exhibit negative temperature behavior (decreasing entropy with increasing energy). This behavior was first experimentally demonstrated by Purcell and Pound in 1951. Our formal definition of temperature in terms of the slope of $S(U)$ came from an analysis of two systems in thermal equilibrium. We can construct analogous relations for other thermodynamic variables by considering other types of equilibrium. For example, a system in which volume can be exchanged via a movable wall will come to mechanical equilibrium when the pressures on the two sides of the wall are equal. Assuming this system is also in thermal equilibrium, we can define pressure in terms of a volume derivative of the entropy as given above. Similarly, we can set up a system with a porous wall allowing for the exchange of particles. This system will come to diffusive or chemical equilibrium when the chemical potentials of the two sides are equal, where chemical potential is defined in terms of a derivative of entropy with particle number. Finally we will apply the second law restrictions on entropy changes to establish limitations of the efficiency of any possible heat engine (or refrigerator). We will analyze several models of real world heat engines including gasoline, diesel, and steam engines.

Problem Assignment: (10 problems total)

- **3.19** (Algebra for the paramagnet analytic solution)
- **3.23** (Entropy-temperature function for the paramagnet)
- **3.25** (Entropy and specific heat for an Einstein solid)
- **3.28** (Entropy change in a constant pressure process)
- ***3.32** (Rapid compression of a gas) **[Ryogo]**
- **3.36** (Chemical potential of an Einstein solid)
- **3.37** (Chemical potential and density variation for the atmosphere)
- **4.1** (Thermodynamic cycle as a heat engine)
- **4.20** (Efficiency of a diesel engine)

***A1**. (Hot block power source) A heat engine is run with a large metal block as the hot reservoir and the ocean as the cold reservoir. The block has initial temperature T_i and heat capacity $C = Mc$, where M is the block mass and c is the metal specific heat. The ocean remains at temperature T_o.
(a) Calculate the maximum amount of work that can be done by this engine. Express your answer in terms of T_i, T_o, and C only.
(b) Obtain a numerical result for a 1000 kg aluminum block with T_i = 600 K. Assume an ocean temperature of 280 K. **[Su]**

*denotes a problem to be presented in class