Thermal Physics: Problem Set #3

Ideal Gas Multiplicity, Entropy, and Temperature

$$\Omega_{gas}(U,V,N)=\frac{1}{N!}\frac{V^N}{h^{3N}}\frac{\pi^{3N/2}}{(3N/2)!}\left(\sqrt{2mU}\right)^{3N} \;;\; S=k\ln\Omega \;;\; \frac{1}{T}=\left(\frac{\partial S}{\partial U}\right)_{N,V} \;;\; dS=\frac{Q_{rev}}{T} \;;\; \Delta S=\int_{T_i}^{T_f}\frac{C_v}{T}dT$$

Due: Friday Jan. 30 by 6 pm

Reading Assignment: for Mon, 2.6, 3.1 (18 pgs) (Ideal gas entropy and Def. of Temperature)
for Wed, 3.2-3.3 (16 pgs) (Measuring entropy and Paramagnetism)
for Fri, 3.4 (8 pgs) (Mechanical equilibrium)

Overview: Although the ideal gas is a simple model, enumerating microstates (i.e., computing the multiplicity) of this model is much more difficult then for the Einstein solid. In a gas the number of microstates depends not only on the number of particles and total energy but also on the system volume. Furthermore, the kinetic energy of the gas molecules is treated as a continuous variable so simple combinatorics cannot immediately be used to count energy arrangements. However, by appealing to quantum mechanics to define a volume element in "phase space", counting states becomes possible leading to the multiplicity function given at the top of the page. Boltzmann's connection between macrostates and microstates is formalized in his famous definition of entropy (given on his tombstone) as the log of multiplicity. The second law of thermodynamics is the statement that an isolated system tends to evolve to the macrostate characterized by the largest number of microstates (which, assuming all microstates are equally likely, is the most probable macrostate). Thus, the second law is really a statement of probability and the most probable or equilibrium state is the state of maximum entropy. At equilibrium a macroscopic system will randomly sample an enormous number of microstates but, on timescales exceeding the age of the universe, will never leave this most probable macrostate. Two systems brought into thermal contact are said to be at equilibrium when their temperatures have become the same. These two definitions of equilibrium (maximum entropy and equal temperature) allow us to construct a formal definition of temperature as the slope of an entropy-energy function. In practice, entropy changes can be measured for a process by measuring the heat flow associated with the process.

Problem Assignment: (10 problems total)

- ***2.28** (Entropy production in card shuffling) **[Richard]**
- **2.29** (Entropy of an Einstein solid)
- **2.31** (Sackur-Tetrode equation ... doing the algebra)
- **2.33** (Entropy of a mole of argon gas)
- ***3.7** (Temperature of a black hole) **[Andrew]**
- **3.10** (Entropy change for a melting ice cube)
- ***3.11** (Entropy change in mixing hot and cold water) **[Christian]**
- **3.14** (Entropy of a cold aluminum block)
- **3.16** (Entropy production in a computer memory)

A1. At low temperature an Einstein solid with large q and N will have $q<<N$ and a multiplicity give approximately by $\Omega \approx (eN/q)^q$. Find an expression for the temperature of this solid as a function of $U = q\varepsilon$ and invert this to show that $U = N\varepsilon\, e^{-\varepsilon/kT}$. Compute the specific heat for this model and plot $C_v/N_{\text{atom}}k$ vs $T^* = kT/\varepsilon$ for $0 \le T^* \le 1.0$ where $N_{\text{atom}}=N/3$. Einstein originally proposed this model to explain the decrease in C_v for solids at low temperatures (see Fig. 1.14).

*denotes a problem to be presented in class