Thermal Physics: Problem Set #2

Multiplicity in Model Systems: Einstein Model

$$\Omega(N,n) = \frac{N!}{n!\,(N-n)!} = \binom{N}{n} \; ; \; \Omega_{\text{Einstein}}(N,q) = \binom{q+N-1}{q} \; ; \; N! \approx N^N e^{-N}\sqrt{2\pi N} \; ; \; \ln N! \approx N \ln N - N$$

Due: Friday Jan. 23 by 6 pm

Reading Assignment:

for Mon,	No Class		(MLK Day)
for Wed,	2.2-2.4	(15 pgs)	(Multiplicity and the Einstein Model)
for Fri,	2.5-2.6	(16 pgs)	(Multiplicity of Ideal Gas and Entropy)

Overview: Why does thermal energy spontaneously flow from hot to cold? This seems like a simple question but it took the physics community a long time to come to an understanding of this everyday behavior. The difficulty here is that the underlying laws of physics are completely "time-reversible" (unchanged under the operation $t \rightarrow -t$) whereas the spontaneous flow of heat is an irreversible process. Boltzmann was the first person to explain how irreversible behavior can arise out of reversible (and random) motion. What Boltzmann realized was that each macroscopic state of a system corresponds to an enormous number of distinct microscopic conformations or microstates. When two systems at different temperatures are brought into thermal contact, completely random microscopic motion will redistribute the energy such that we always observe energy moving from hot to cold ... even though the basic laws of physics would not prohibit the opposite from occurring. To get more insight into this issue we will study some elementary combinatorics which allows us to enumerate microstates for a given macrostate. Then we will apply this combinatoric analysis to the Einstein model of a solid. Einstein's model treats each atom in a solid as three independent quantum-mechanical oscillators. Stirling's approximation for the evaluation of factorials helps to make some of the numerical analysis involving large and "very-large" numbers possible.

Problem Assignment: (9 problems total)

1.34 (A thermodynamic cycle)
2.1 (Combinatorics: 4 coins)
*__2.2__ (Combinatorics: 20 coins) **[Su]**
2.5 a,b,d (Einstein model microstates)
2.8 (Einstein solids in thermal contact)
2.9+2.10 (Einstein solids in thermal contact - Use StatMech program ... you need to select "Count Oscillators" from the Options menu. Submit tables and graphs for 2.9, annotated graph only for 2.10)
2.18 (Stirling's approximation for Einstein solid multiplicity)
*__2.22__ (Width of multiplicity function for two large Einstein solids) **[Ryogo]**
2.23 (Microstate dynamics of a paramagnet and the meaning of "very-large" numbers)

Bonus: (More elementary probability)
a. What is the probability of throwing a total of six points or less with 3 dice?
... Now consider throwing 6 dice. Find the probability of obtaining:
b. exactly one ace; **c**. at least one ace; **d**. exactly two aces (snakes eyes!).

*denotes a problem to be presented in class